DYNAMICS OF PLANE FRAMES SUBJECTED TO VERTICAL SUPPORT MOTION

-by-

S.Z.H. Burney* and L.G. Jaeger**

INTRODUCTION

In the design of structures subjected to seismic loading it is usually assumed that such perturbations have a predominantly horizontal component. It is further assumed that the vertical component is negligible, or else its effect is combined with that of the horizontal contribution in an arbitrary manner. Very little is known about the response of structures when subjected to fluctuating vertical support motion. Therefore, before one can attempt to study the response of structures to the combined effects of horizontal and vertical motions it is necessary to resolve the problem of vertical support motion. As a preliminary to attacking the problem of seismic perturbations, which are of a random nature, it is required to seek solutions for cases of periodic support motion.

Jaeger and Barr [1,2] investigated the problem of the dynamic stability of a cantilever column subjected to periodic vertical support motion and showed that this class of problem falls in the category of "parametric" instability problems. Burney and Jaeger [3] have presented a method of obtaining the regions of dynamic instability of such a cantilever by a numerical method approach.

This paper discusses the parametric instability of plane frames. A method is proposed to obtain the regions of dynamic instability. An experimental investigation was also carried out and the results obtained are presented.

THE PROPOSED METHOD

The floor system of the frame is idealised as a continuum. Consider the frame shown in Fig. 1. Let axial changes of length of the columns be ignored and let the actual horizontal beams be replaced by a medium of bending stiffness $\frac{NEI}{H}_T$ per unit height.

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Consider a slice, of height dx, of the medium at height x above the base. Let the end moment per unit height be called m. From Fig. 2 and using the moment area method we have:

$$\frac{1}{2}(m\,dx)\frac{b}{2}\,\frac{2}{3}\,b \;=\; \frac{(NEI_T)}{H}\,dx\;b\;\frac{dy}{dx}$$

$$\text{So} \qquad m \;=\; \frac{6NEI_T}{Hb}\,\frac{dy}{dx} \qquad \text{.........(1)}$$

Let q(x) be a static external force acting per unit height on the left hand column. Considering the equilibrium of the element shown in Fig. 3 the following relations are obtained:

$$\frac{dV}{dx} \;=\; -\,\frac{q}{2} \qquad \text{.........(2)}$$

$$V \;=\; \frac{dM}{dx} + m \qquad \text{.........(3)}$$

Differentiating Eq. (3) w.r.t. x and eliminating V gives:

$$\frac{d^2M}{dx^2} + \frac{dm}{dx} \;=\; -\,\frac{q}{2} \qquad \text{.........(4)}$$

Now $M = -\,EI\dfrac{d^2y}{dx^2}$; consider the case $q(x) \;=\; \dfrac{q_o x}{H}$ then we get:

$$\frac{d^2M}{dx^2} - \alpha^2 M \;=\; -\,\frac{q_o x}{2H} \qquad \text{.........(5a)}$$

$$\text{or} \quad \frac{d^4y}{dx^4} - \alpha^2\,\frac{d^2y}{dx^2} \;=\; \frac{q_o x}{2EIH} \qquad \text{.........(5b)}$$

$$\text{where} \quad \alpha^2 \;=\; \frac{6NEI_T}{HbEI} \qquad \text{.........(6)}$$

The solution of Eq. (5) may be obtained readily as:

$$M \;=\; A\sinh\alpha x + B\cosh\alpha x + \frac{q_o x}{2H\alpha^2} \qquad \text{.........(7)}$$

Satisfying the following boundary conditions:

(i) at $x = o$, $y = o$

(ii) at $x = o$, $\dfrac{dy}{dx} = o$

(iii) at $x = H$, $M = o$
at $x = H$, $V = o$

yields the following expression:

$$y = \frac{q_0}{2H\alpha^3 EI}\left\{(1 - \frac{\beta^2}{2})(\sinh\frac{\beta x}{H} - \frac{\beta x}{H}) + \frac{[\beta - (1 - \beta^2/2)\sinh\beta]}{\cosh\beta}(\cosh\frac{\beta x}{H} - 1) - \frac{(\frac{\beta x}{H})^3}{6}\right\} \quad \ldots\ldots\ldots(8)$$

where $\beta = \alpha H$

Equation (8) gives the deflected shape of the frame and is a function of β, a measure of the relative stiffness of the beams w.r.t. the columns. The case $\beta = o$ represents a free cantilever column.

The modal shape for the frame shown in Fig. 4 was determined using Eq. (8) and is shown in Fig. 5 by the solid line. The modal shape was also obtained using the finite element method and is indicated by triangular markers in Fig. 5, from which we conclude that the continuum approach gives a good prediction to the modal shape for a portal frame.

Having developed an expression for the deflected shape of a portal frame we are now in a position to study its dynamic instability. Let the vertical support motion be $u_s(t)$. Considering the vertical motion of the element shown in Fig. 6 yields the following relations:

$$P = -\int_x^H \rho_o \ddot{u}_s \, dx \quad \ldots\ldots\ldots(9)$$

where ρ_o = mass per unit height for half of the structure.

$$\frac{dV}{dx} = -\frac{q}{2} \quad \ldots\ldots\ldots(10)$$

and $$V = \frac{dM}{dx} - P(x)\frac{dy}{dx} + m \quad \ldots\ldots\ldots(11)$$

Differentiating Eq. (11), substituting for V and m gives:

$$\frac{d^4y}{dx^4} + \frac{\rho_o}{EI}(H - x)\ddot{u}_s\frac{d^2y}{dx^2} - \frac{\rho_o}{EI}\ddot{u}_s\frac{dy}{dx} - \alpha^2\frac{d^2y}{dx^2} = \frac{q(x)}{2EI}$$

now $$q(x) = -\rho\frac{\partial^2 y}{\partial t^2} \quad \ldots\ldots\ldots(11a)$$

where ρ = mass per unit height for the whole structure i.e. $\rho = 2\rho_o$

Substituting in the above equation we have:

$$\frac{\partial^4 y}{\partial x^4} + \frac{\rho}{2EI}(H - x)\,\ddot{u}_s \frac{\partial^2 y}{\partial x^2} - \frac{\rho}{2EI}\ddot{u}_s \frac{\partial y}{\partial x}$$

$$- \alpha^2 \frac{\partial^2 y}{\partial x^2} + \frac{\rho}{2EI}\frac{\partial^2 y}{\partial t^2} = o \qquad \text{.........(12)}$$

With the help of Eq. (8) it may be shown that:

$$\frac{\partial^4 y}{\partial x^4} - \alpha^2 \frac{\partial^2 y}{\partial x^2} = R\alpha^5 x \qquad \text{.........(13)}$$

$$\text{where} \quad R = \frac{q_o H^4}{2EI\beta^5} \qquad \text{.........(14)}$$

Equation (12) then writes as:

$$\frac{\rho}{2EI}(H - x)\,\ddot{u}_s \frac{\partial^2 y}{\partial x^2} - \frac{\rho}{2EI}\ddot{u}_s \frac{\partial y}{\partial x} + R\alpha^5 x$$

$$+ \frac{\rho}{2EI}\frac{\partial^2 y}{\partial t^2} = o \qquad \text{.........(12a)}$$

$$\text{Let} \quad y(x,t) = \sum_{i=1}^{\infty} R\,\phi_i(x) f_i(t) \qquad \text{.........(15)}$$

where ϕ_i (x) are a complete set of functions satisfying the boundary conditions, for instance the various modes of free vibration of the problem, and f_i(t) are generalized co-ordinates. An approximate solution for y may be obtained by curtailing the series at some integer $i = n$. This approximation will not satisfy Eq. (12a), but the error introduced by the substitution of the curtailed series may be minimized by applying the Galerkin Method [4].

$$\int_o^H R\Big[\frac{\rho}{2EI}(H - x)\,\ddot{u}_s \sum_{i=1}^{n} f_i \phi_i'' - \frac{\rho}{2EI}\ddot{u}_s \sum_{i=1}^{n} f_i \phi_i' + \alpha^5 \sum_{i=1}^{n} f_i$$

$$+ \frac{\rho}{2EI}\sum_{i=1}^{n} \ddot{f}_i \phi_i\Big]\phi_j\,dx = o \quad (i = 1,2, \ldots n) \qquad \text{.........(16)}$$

Let us consider a one term series approximation for y, i.e. $y = R\,\phi_1(x) f_1(t)$.

Equation (16) then writes as:

$$\int_o^H \left[\left\{ \frac{\rho}{2EI} (H - x)\, \ddot{u}_s \frac{d^2\phi_1}{dx^2} - \frac{\rho}{2EI} \ddot{u}_s \frac{d\phi_1}{dx} + \alpha^5 x \right\} f_1(t) \right.$$

$$\left. + \frac{\rho}{2EI} \frac{d^2 f}{dt^2} \phi_1 \right] \phi_1 dx = o$$

or

$$\int_o^H \left[\left\{ \frac{\rho}{2EI} (H - x)\, \ddot{u}_s \phi_1 \frac{d^2\phi_1}{dx^2} - \frac{\rho}{2EI} \ddot{u}_s \phi_1 \frac{d\phi_1}{dx} \right.\right.$$

$$\left. \left. + \alpha^5 x \phi_1 \right\} f_1(t) + \frac{\rho}{2EI} \phi_1^2 \frac{d^2 f}{dt^2} \right] dx = o \qquad \text{.........(17)}$$

To solve Eq. (17) we have first to evaluate the integrals

$$\int \phi_1 \frac{d^2\phi_1}{dx^2} dx, \int x \phi_1 \frac{d^2\phi_1}{dx^2}, \int \phi_1 \frac{d\phi_1}{dx} dx, \int x \phi_1 dx, \int \phi_1^2 dx.$$

From Eq. (8) we have:

$$\phi_1(x) = \frac{\gamma_o}{2H\alpha^5 EI} \left\{ (1 - \frac{\beta^2}{2})(\sinh \frac{\beta x}{H} - \frac{\beta x}{H}) + \frac{\left[\beta - (1 - \frac{\beta^2}{2}) \sinh\beta\right]}{\cosh\beta} \right.$$

$$\left. (\cosh \frac{\beta x}{H} - 1) - \frac{(\beta \frac{x}{H})^3}{6} \right\} \qquad \text{.........(8a)}$$

Equation (8a) and its derivatives may be used to evaluate numerically the above integrals.

Equation (17) is a Mathieu equation and its solution would yield the region of dynamic instability.

The frame shown in Fig. 4 was analyzed by this procedure. Its mass was found to be 0.465 lbs. Substituting the value of the integrals in Eq. 17 yields

$$21.8826 \frac{d^2 f}{dt^2} + (17600 - 0.9805\, \ddot{u}_s)\, f(t) = o$$

Letting $u_s = C \cos \Omega t$ results in

$$21.8824 \frac{d^2 f}{dt^2} + (17600 + 0.9805 \ \Omega^2 C \cos \Omega t) f = o$$

The solution of this Mathieu equation gives

$$\Omega^2 = \frac{3230}{1 \pm 0.09C} \quad ....(18)$$

Equation (18) gives the value of the disturbing frequency required to cause the frame to go into instability in the first mode.

EXPERIMENTAL PROCEDURE

An experimental investigation of the parametric instability of portal frames was carried out. A six storey portal frame as shown in Fig. 4 was subjected to vertical support motion.

The frame was constructed from aluminum strips 3/4" wide and 0.040" thick. The same section was used for columns and beams. The beams were spaced at 6" centre to centre and were bolted to the columns by means of 1/2" x 1/2" x 1/16" aluminum brackets as shown in Fig. 4. The frame was clamped to an aluminum plate 5-1/4" x 3" x 3/8" which was directly mounted on the shaker.

A schematic diagram of the experimental set-up is shown in Fig. 7. A sinusoidal waveform is generated by the function generator and amplified by the amplifier to drive the shaker, a Goodman's V-50 model. The frequency and amplitude of the oscillation of the base motion were measured by a Sanborn displacement transducer (D.C. excited). Fig. 8 shows the apparatus used to conduct the experiment.

As the investigation was of a qualitative nature i.e. it was intended only to ascertain whether the motion was stable or unstable, a visual criterion was used. The motion was termed unstable when the structure departed markedly from its initial configuration. The frame was constructed from aluminum to enable the development of the instability modes to be visually detected. From the photographs of the instability modes as shown in Fig. 9 to Fig. 12 it may be seen that it is fairly easy to recognize an instability mode.

The amplitudes could only be measured for the first three modes (shown in Fig. 9 to Fig. 11). For the fourth mode (Fig. 12) the amplitudes were so small that they could not be detected by the deflection transducer. The photographs of Fig. 9 to Fig. 12 are for the natural modes corresponding to the principal region of instability. The instabilities corresponding to the first secondary region (i.e. the second region of instability) were also detected visually but the amplitudes were small and could not be recorded.

The results of the experimental investigation are shown in Fig. 13 to Fig. 15.

The natural frequencies for the first four modes in c.p.s. as estimated by the finite element method and as obtained experimentally were as follows:

Mode	1	2	3	4
Finite Element	3.83	12.75	22.77	34.33
Experimental	3.80	13.7	23.5	36.0

DISCUSSION OF RESULTS

From Eq. (18) Ω = 57.0 rad./sec. when C = o. However, from experimental results the value of the disturbing frequency, Ω , corresponding to the least amplitude of the base motion was 47.8 rad./sec. Equation (18) does not give a good prediction of the width of the instability region. This is possibly due to the fact that the above analysis considered only a one term series approximation for y. If more terms are included a better estimate of the width of the instability region would be obtained.

CONCLUSIONS

The continuum approach may be used to obtain the region of dynamic instability of portal frames. As shown above, the method gives a good estimate of the deflected shape and the natural frequency of the first mode of the structure. However, the prediction of the width of the instability zone is not satisfactory.

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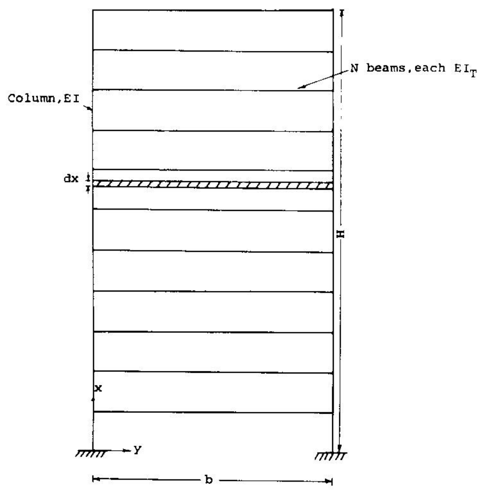


Fig. 1: Multi-storey portal frame.

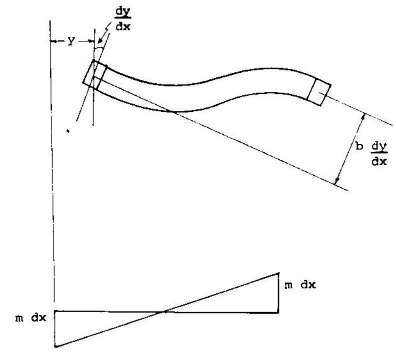


Fig. 2: Bending moment diagram for strip of medium.

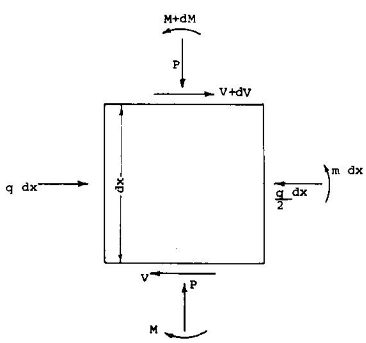


Fig. 3: Forces on an element of the column.

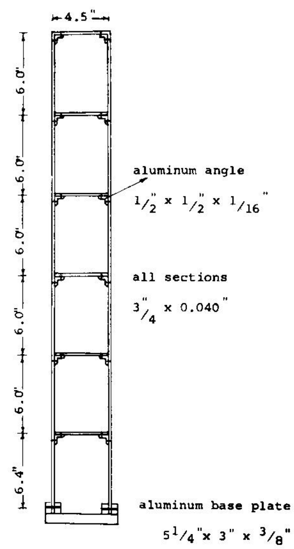


Fig. 4: Six-storey portal frame.

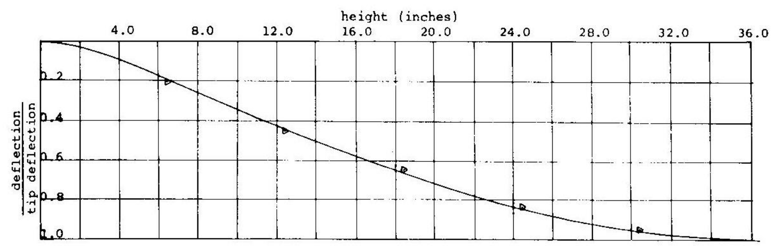


Fig. 5: Comparison of deflection by continuum and finite element methods.

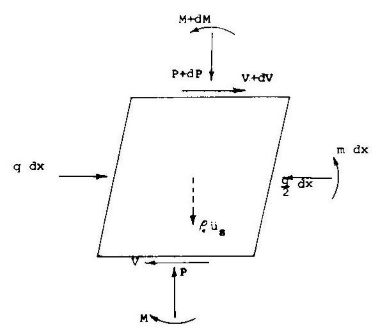


Fig. 6: Forces on an element of the column.

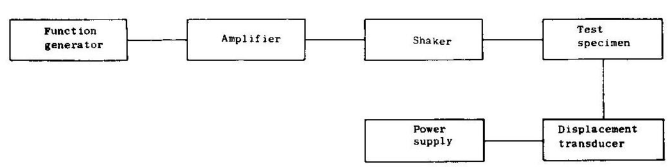


Fig. 7: Schematic diagram of the experimental set-up.

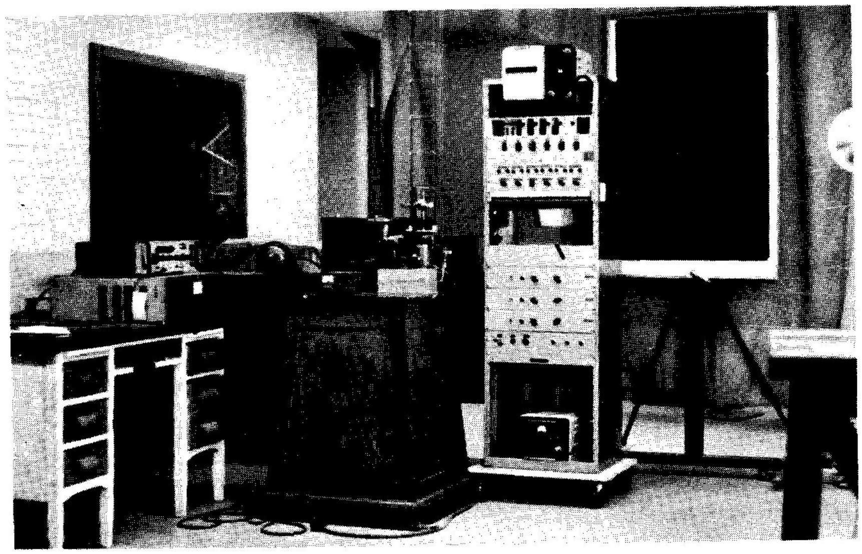
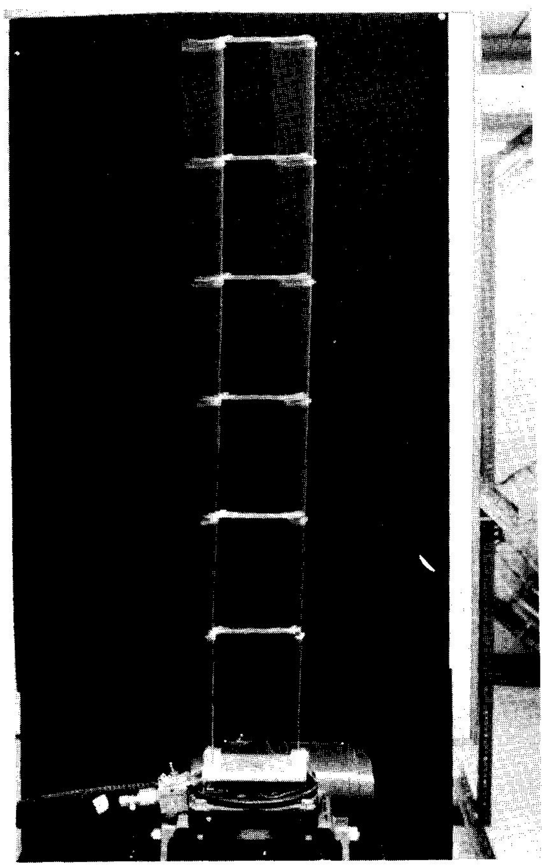


Fig. 8: Experimental set-up



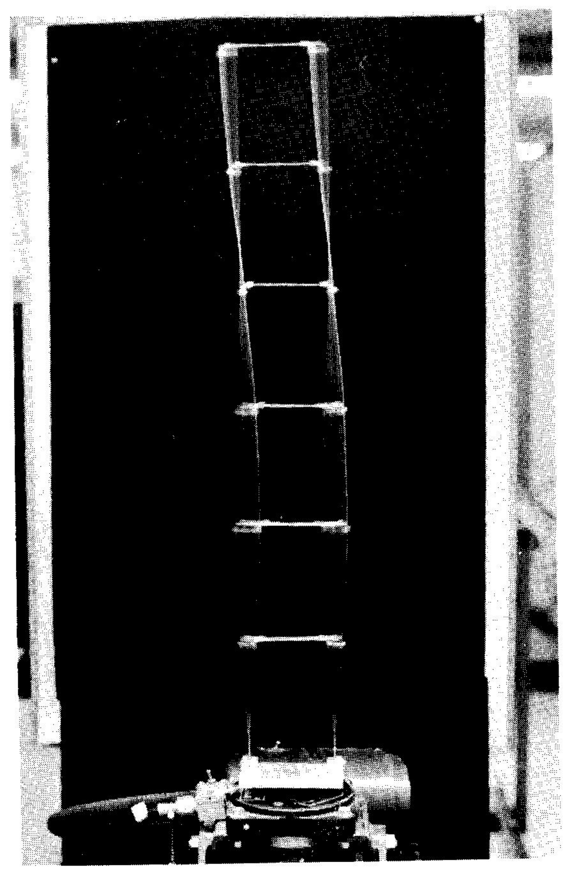


Fig. 9: First mode instability. Fig. 10 Second mode instability.

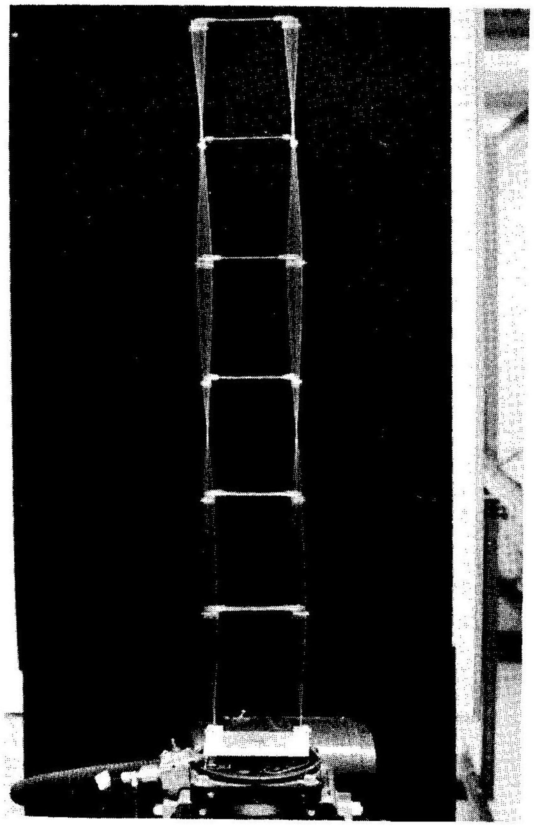


Fig. 11: Third mode instability.

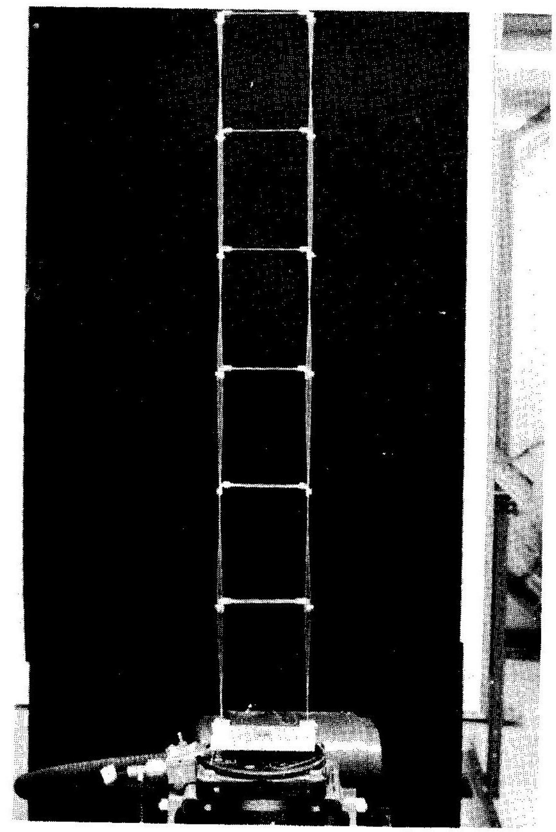


Fig. 12: Fourth mode instability

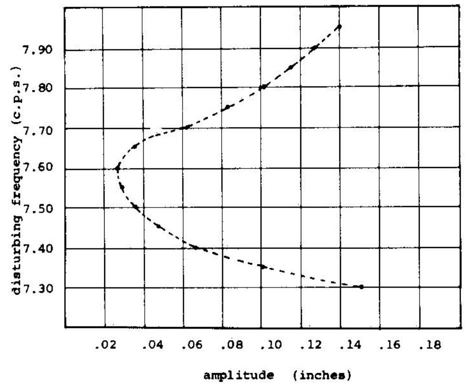


Fig. 13: Principal instability region for first mode.

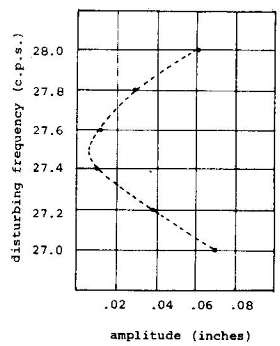


Fig. 14: Principal instability region for second mode.

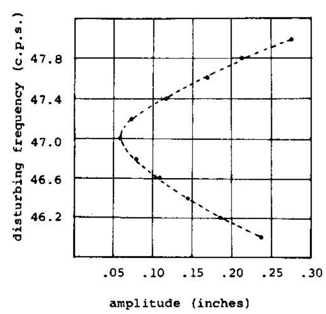


Fig. 15: Principal instability region for third mode.

DISCUSSION OF PAPER NO. 12

DYNAMICS OF PLANE FRAMES SUBJECTED TO VERTICAL SUPPORT MOTION

by

S. Z. H. Burney and L. G. Jaeger

Discussion by: W. K. Tso

Experience on the study of vertical earthquake motion on a single degree of freedom system in the form of an inverted pendulum leads the discusser to believe the problem of parametric resonance of buildings due to vertical earthquake motion is not important. The study carried out at McMaster consisted of applying simultaneously the vertical and horizontal components of 1940 El Centro earthquake to the vertical pendulum with elastic hinges. It was found that the response is almost the same as those obtained with horizontal excitation only. This observation leads the discusser to believe, while parametric resonance is possible due to sinusoidal vertical motion at the base, as shown in the paper, parametric resonance is not likely to be an important consideration as far as structural response subject to vertical earthquake motion is concerned.

Discussion by: A. Heidebrecht

The phenomenon of parametric instability described by the authors does exist and can be demonstrated theoretically and experimentally. However, this paper would not seem to add much to the knowledge of this phenomenon, since the experimental phenomenon was demonstrated by Guru and Heidebrecht at CANCAM 1969 (Waterloo) and Guru has recently described the analysis in a paper published in the 1970 Rumanian Conference on Earthquake Engineering. Many other authors in the dynamic instability field have written about the nature of the regions of instability in such situations. Also, as Dr. W. K. Tso has already indicated in his discussion of this paper, it would not seem that this phenomenon is significant in the response of structures subjected to both horizontal and vertical excitation in an earthquake. Both the short duration of excitation and its randomness would contribute to the resistance to this type of instability.

Discussion by: O. A. Pekau

The speaker has implied that parametric excitation may be of importance to buildings subjected to earthquake ground motions. On this point, I wish to refer to work at the California Institute of Technology conducted by Husid in 1967. For simple structures subjected simultaneously to horizontal as well as vertical earthquake motions, he found that the effect on lateral response was quite small. Indeed, the action of vertical support motion did not necessarily increase response, suggesting then that the phenomenon of parametric excitation may be of no interest to earthquake engineers.

Question by: M. Novak

The authors seem to suggest that the parametric instability of frames might arise due to vertical component of seismic motion. I wonder if they could give any indications concerning (1) the magnitude of amplitude of vertical vibrations necessary to trigger the dynamic instability in a typical building (the inclusion of damping is necessary to get any idea about that); (2) the time required to develop the required amplitude with random excitation (the first passage problem). These aspects would help to evaluate the possibility for the parametric instability to happen.

Could the authors indicate what the frequencies were of excitation and response shown in the movie?

What checks were used to prove that the motion observed was the parametric instability and not just bending resonant oscillations? It was difficult to recognize it in the movie.

Question by: S. R. Swanson

When the electromagnetic shaker moves up and down, it also moves sideways due to the flexure installation of the driver coil. Could this be an instability triggering mechanism?

Combined Reply to above Discussions and Questions by: S. Z. H. Burney and L. G. Jaeger

The phenomenon of the parametric excitation of structures has been experimentally observed in the laboratory by the authors and many others. Utida and Sezawa(1), Bolotin(2), Somerset and Ewan-Iwanowski(3) and Weingarten(4) have demonstrated it experimentally for the case of columns, while Guru and Heidebrecht(5) and Barr and McWhannel(6) observed it for the case of portal frames. The authors also undertook an experimental investigation into the parametric instability of portal frames in 1967; about this time work was going on in this general area at several universities, for example, McMaster and Edinburgh.

Ward(7) made a study of 19 different building codes and observed that 7 of them include the effect of vertical accelerations by increasing the weight of the building from 7 to 40%. In the opinion of the authors, before the effect of vertical accelerations may be dismissed as trivial, a more thorough investigation of the response of structures to simultaneous vertical and horizontal support motions is required. It is likely that the magnitude of the effect of vertical accelerations depends upon the degree of cross correlation between the horizontal and vertical acceleration regimes, in the case of random excitation.

The present investigation was only one aspect of understanding the response of structures to vertical support motion. Hence an attempt was not made to predict the required amplitude of the support motion to trigger off instability for the case of actual buildings. An experimental investigation of the effects of damping on the parametric response of columns was carried out by Somerset and Ewan-Iwanowski(3).

In the movie shown, the frequencies of the support motion and that of the structure are given in the paper, e.g. for the principal instability region instability will occur when the exciting frequency is twice the natural frequency of the structure.

The motion of the shaker was verified to be truly vertical, without horizontal or rocking motions, by means of displacement transducers.

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